Geometry Review

Draw a right triangle. What makes a triangle a right triangle?

![Diagram of a right triangle with labels for hypotenuse and legs.]

What do we call the sides of right triangle? Label them in your diagram.

- legs
- hypotenuse

What types of angles does a right triangle have?

- one right angle
- two acute angles

State the Pythagorean Theorem?

\[ a^2 + b^2 = c^2 \]

**Example 1:** Use the Pythagorean Theorem to find \( AB \).

![Diagram with labels for sides 7, 15, and \( AB \).]

Now, remember back to your days in geometry... What are the names of the three ratios we can use to find the missing side lengths or missing angles in a right triangle?

1. Sine

2. Cosine

3. Tangent
Right Triangle Trigonometry

![Triangle Diagram]

Use the above diagram to answer the following questions:

1. What side is opposite \( \angle A \)?
   - BC

2. What side is adjacent to \( \angle B \)?
   - BC

Describe what it means for a leg to be adjacent to an angle in a right triangle.

A leg is adjacent to an angle if it is one of the two sides that form the angle with the hypotenuse.

What does SohCahToa mean?

- Soh means sine is the opposite leg over the hypotenuse
- Coh means cosine is the adjacent leg over the hypotenuse
- Toa means tangent is the opposite leg over the adjacent leg

**Example 2:**

![Right Triangle 2]

\[ \sin B = \frac{3}{5} \]

What is the value of \( \sin(B) \)?

- A) \( \frac{4}{5} \)
- B) \( \frac{3}{4} \)
- C) \( \frac{3}{5} \)
- D) \( \frac{3}{5} \)
Example 3:

\[
\begin{align*}
\sin A &= \frac{4}{5} \\
\cos A &= \frac{3}{5}
\end{align*}
\]

Determine whether each equation is correct. Select Yes or No for each equation.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sin(A) = \frac{1}{5}</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>\cos(A) = \frac{3}{5}</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>\sin(B) = \frac{1}{5}</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>\cos(B) = \frac{1}{4}</td>
<td>✔</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sin B &= \frac{3}{5} \\
\cos B &= \frac{4}{5}
\end{align*}
\]

Example 4: What is the value of \( \sin B \)?

\[\sin B = \frac{3}{5}\]

Example 5: Triangle ABC is similar to triangle WYZ.

\[
\begin{align*}
\angle A &= \angle W \\
\angle B &= \angle Y \\
\angle C &= \angle Z
\end{align*}
\]

Select all angles whose tangent equals \( \frac{3}{4} \).

- A) \( \angle A \)
- B) \( \angle B \)
- C) \( \angle C \)
- D) \( \angle W \)
- E) \( \angle Z \)
- F) \( \angle Y \)
Consider the triangle to the right.

What is the value of $\cos B$? \( \frac{4}{5} \)

What is the value of $\sin A$? \( \frac{4}{5} \)

What seems to be true about the sine of one acute angle and the cosine of the other acute angle in a right triangle?

\[
\sin A = \cos B
\]

**Example 6:** Assume we have a right triangle $ABC$ where $A$ and $B$ are the acute angles. Let $\sin(47^\circ) = 0.7314$. What would the measure of $\angle B$ have to be so that $\cos B = 0.7314$?

\[
\sin A = \cos B
\]

Determining $\angle B$:

\[
90 - 47 = 43^\circ
\]

**Example 7:** Let $\sin(30^\circ) = \frac{1}{2}$. What would $m\angle B$ need to be so that $\cos(B)$ is $\frac{1}{2}$?

\[
m\angle B = 60^\circ
\]

\[
90 - 30 = 60
\]

**Example 8:** Triangle $ABC$ is similar to triangle $WYZ$.

\[
s^2 + 12^2 = (WY)^2
\]

\[
WY = 13
\]

Determine whether each statement is true. Select True or False for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin(A) &lt; \sin(Y)$</td>
<td>✓</td>
<td>✔</td>
</tr>
<tr>
<td>$\cos(B) = \sin(W)$</td>
<td>✓</td>
<td>✔</td>
</tr>
<tr>
<td>$\tan(W) &gt; \tan(A)$</td>
<td>✔</td>
<td>✓</td>
</tr>
</tbody>
</table>

\[
\sin A = \frac{24}{26}, \quad \sin Y = \frac{5}{12}
\]

\[
\cos B = \frac{24}{26}, \quad \sin W = \frac{12}{13}
\]

\[
\tan W = \frac{12}{5}, \quad \tan A = \frac{24}{10}
\]
Example 9: Consider this right triangle.

\[ \begin{array}{c|c|c} 
\text{Yes} & \text{No} \\
\hline 
13\sin(\theta) & X & X \\
13\cos(A) & X & X \\
12\tan(A) & X & X \\
12\tan(\beta) & X & X \\
\end{array} \]

Determine whether each expression can be used to find the length of \( AC \).
Select Yes or No for each expression.

When using your calculator to solve right triangle trigonometry problems, make sure it is in degree mode!!!

When would you use the second button for \( \sin^{-1} \), \( \cos^{-1} \), \( \tan^{-1} \) button?

\[ \sin \rightarrow \cos \rightarrow \tan \rightarrow \text{allow us to find angle measures} \]

Example 10: Consider the right triangle. Determine the length of \( AC \) to the nearest tenth.

\[ \text{AC ? opp} \]
\[ 15 \text{ hyp} \]
\[ \sin 64^\circ = \frac{AC}{15} \]
\[ 15 \sin(64^\circ) = AC \]
\[ 13.48 = AC \]

13.5
**Example 11:** Consider the right triangle. What is the measure of $\angle A$, to the nearest degree?

\[
\begin{align*}
\sin \theta &= \frac{12}{15} \\
\theta &= \sin^{-1} \left( \frac{12}{15} \right) \\
\theta &\approx 53^\circ
\end{align*}
\]

**Example 12:** Bob uses a 20 foot ladder to paint a section of his house that is 16 feet high.

\[
\sin \theta = \frac{16}{20} \quad 16^2 + b^2 = 20^2 \quad b = 12
\]

Select all equations that can be used to solve for $\theta$.

- A. $\sin \theta = \frac{12}{20}$
- B. $\cos \theta = \frac{12}{20}$
- C. $\tan \theta = \frac{12}{16}$
- D. $\sin \theta = \frac{14}{20}$
- E. $\cos \theta = \frac{14}{20}$
- F. $\tan \theta = \frac{14}{20}$
**Example 13:** Donna wants to calculate the height of a tree. She makes the following measurements.

- The length of the tree’s shadow is 29 meters.
- The angle of elevation from the ground to the top of the tree is 30°.
- The tree stands perpendicular to the ground.

![Diagram of a tree and its shadow]

What is the height of the tree, in meters? Round your answer to the nearest whole meter.

\[
\tan 30° = \frac{BC}{29}
\]

\[
29 \tan 30° = BC = 17
\]

Fundamentals of solving a trig problem:
1) You need three things: 2 known and 1 unknown.
2) You always start at an angle! And then name sides in relation to that angle.
3) Which side of the triangle never changes its name – it is secure in its identity? **Hypotenuse**
4) Which two sides change dependent upon which angle you start from? **Opposite leg & adjacent leg**
5) Note: Never start from the right angle.

**Definitions**

- **Angle of depression**
  - Illustrated:

- **Angle of elevation**
  - Illustrated:
When looking at a unit circle the cosine value is always the $x$ value and the sine value is always the $y$ value. (They go in alphabetical order)

$$\left( \cos, \sin \right)$$

For example,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}.$$  

Another example is $\cos 120^\circ = \frac{-1}{2}$.